Table 2 Weighted disturbance

Trip designation	Weighted value
AB, BA	$+1\delta$, -1δ, respectively
AC, CA	$+3\delta$, -3δ, respectively
BC, CB	$+2\delta$, -2δ, respectively

interrupt the photocell beam by hand waving or to and fro motion without actually taking a trip. Thus, improved techniques using a greater number of photocells and possibly incorporating television monitoring would help in upgrading the trip data resulting from future experiments.

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Simple Formulas for Unsteady Pressure on Slender Wedges and Cones in Hypersonic Flow

K. J. Orlik-Rückemann*
National Aeronautical Establishment, Ottawa, Canada

INVISCID surface pressure on slender wedges and cones performing small-amplitude pitching or plunging oscillation has been calculated by McIntosh,[1,2] using the hypersonic small-disturbance theory. Similar information but for slender wedges performing pitching oscillation with an amplitude of the order of the wedge semiangle has been obtained by Kuiken.[3,4] However, the results of all this work are not too readily available; McIntosh's results are in the form of graphs of the static and dynamic lift and pitching moment derivatives vs the inverse of the hypersonic similarity parameter K and also, for wedges only, in the form of an infinite series, whereas Kuiken's results are given as closed-form expressions, but containing functions of K which are only available in the form of a table. In the present Note simple formulas are presented with which the inviscid surface pressures at low frequencies in all the aforementioned cases can be rapidly calculated. It is also shown that for small amplitudes, the error introduced by omission of higher-order terms in frequency is generally less than 1% for reduced frequencies as high as 0.3.

Slender Wedge—Small Amplitude (Any γ)

Let us consider surface pressure on a wedge as the sum of a mean pressure $\bar{p}_m$ associated with the wedge semiangle θ_w and an unsteady oscillatory pressure $\bar{p}_{osc}$ caused by the oscillation of the wedge. The two pressure components may be obtained from the analysis of Ref. 1. Denoting by U and L the upper and the lower surface of the wedge, respectively, we have

$$\bar{p}_m = \bar{p}_\infty(2\gamma K^2 - \gamma + 1)/(\gamma + 1) \tag{1a}$$

$$[\bar{p}_{osc}]_{U,L} = \mp\bar{p}_\infty\gamma M_\infty{}^2\tau\bar{\alpha}e^{ik\bar{t}}A \times \left\{F_1 + 2e^{-ik\bar{x}}\sum_{n=1}^{\infty}(-\lambda)^n e^{ik\Gamma^n\bar{x}}F_2\right\} \tag{1b}$$

where $\bar{p}_\infty$ is the freestream pressure, γ is the specific heat ratio, M_∞ and U_∞ are the freestream Mach number and velocity, τ is the unperturbed shock wave angle, $\bar{\alpha}$ is the amplitude of oscillation, $k = \omega c/U_\infty$ is the reduced frequency based on the wedge chord c, $\bar{t} = tc/U_\infty$ is time, $\bar{x} = xc$ is the axial distance from the leading edge, and A, λ, and Γ are functions of K and γ as follows:

$$A = \{[2\gamma K^2 - \gamma + 1]/[(\gamma - 1)K^2 + 2]\}^{1/2}$$

$$\lambda = [(K^2 + 1)A - 2K^2]/[(K^2 + 1)A + 2K^2]$$

$$\Gamma = (A - 1)/(A + 1)$$

K is the hypersonic similarity parameter based on the unperturbed shock wave slope, that is, $K = M_\infty\tau$. For a wedge performing pitching oscillation around $x = x_0$, we have

$$F_1 = 1 + ik(x - x_0),\ F_2 = 1 + ik(\Gamma^n x - x_0)$$

whereas for a wedge performing plunging oscillation

$$F_1 = ik,\ F_2 = ik$$

Expanding the exponential functions in Eq. (1b), retaining only terms to the order of k^3, and performing the indicated summations, the unsteady oscillatory pressure on a wedge, which performs pitching oscillation $\theta = \theta_0 e^{i\omega\bar{t}} = \bar{\alpha}e^{ik\bar{t}}$ around $x = x_0$ (where positive θ reduces the inclination of the upper surface to the flow), may be written, for $\theta \ll \tau$,

$$[\bar{p}_{osc}]_{U,L} = \mp\bar{p}_\infty\gamma M_\infty AK\{\theta[B_1 + xk^2(xB_4 + x_0B_5)] + \dot{\theta}[xB_2 + x_0B_3 + x^2k^2(xB_6 + x_0B_7)](c/U_\infty)\} \tag{2}$$

The corresponding expression for a wedge that performs plunging oscillation $h = h_0e^{i\omega\bar{t}} = \bar{\alpha}e^{ik\bar{t}}$ (where hc is displacement positive down) is, for $h \ll \tau$,

$$[\bar{p}_{osc}]_{U,L} = \pm\bar{p}_\infty\gamma M_\infty AK\{hxk^2B_5 + \dot{h}[B_3 + x^2k^2B_7](c/U_\infty)\} \tag{3}$$

It may be noted that Eq. (3) can be obtained directly from Eq. (2) by setting, in Eq. (2), $x_0 \gg 1$ (whereas $0 \leq x \leq 1$), $\theta = -h/x_0$, and $\dot{\theta} = -\dot{h}/x_0$.

Functions $B_1, B_2, \ldots, B_7$ depend only on K and γ and are

$$\begin{aligned} B_1 &= 1 - 2C_0,\ B_2 = 1 + 2C_0 - 4C_1 \\ B_3 &= -1 + 2C_0 = -B_1,\ B_4 = C_0 - 4C_1 + 3C_2 \\ B_5 &= 2C_0 - 2C_1,\ B_6 = -\tfrac{1}{3}C_0 + 2C_1 - 3C_2 + \tfrac{4}{3}C_3 \\ B_7 &= -C_0 + 2C_1 - C_2 \end{aligned} \tag{4}$$

where

$$C_n = \lambda\Gamma^n/(1 + \lambda\Gamma^n) \qquad (n = 0,1,2,3)$$

Table 1 Constants for Eq. (9)

Function	a	b	c	d	e
R, $\gamma = \frac{7}{5}$	2.6667	2.4794	1.3733	1.8144	−0.0204
S, $\gamma = \frac{7}{5}$	−2.0000	−1.8055	0.0001	0.7599	−0.1290
R, $\gamma = \frac{5}{3}$	2.2856	2.5128	1.3700	1.4511	0.0163
S, $\gamma = \frac{5}{3}$	−2.0000	0.1725	0.0020	−0.2863	0.0165

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* Head, Unsteady Aerodynamics Laboratory. Associate Fellow AIAA.

Table 2 Maximum relative errors due to curve fitting and omission of higher terms

	R_0	R_1	R_2	R_3
Curve fitting, %	0.008	0.012	0.017	0.080
Omission of higher terms, %		$8.5k^2$	$1.2k^2$	$3.0k^2$

The frequency-independent part of Eq. (2) agrees with Eq. (14) in Ref. 5.† Also, the frequency-independent parts of pressure due to θ and $\dot{h}$ are equal, as they should, since

$$B_3\dot{h}c/U_\infty = -B_1\theta \tag{5}$$

We have the following limiting cases:

$K \rightarrow 1$

$$A = 1,\ \lambda = 0,\ \Gamma = 0 \qquad \text{(for any } \gamma)$$

$K \rightarrow \infty$

$$A = 2.24,\ \lambda = 0.0566,\ \Gamma = 0.383 \qquad (\gamma = \tfrac{5}{3})$$

$$A = 2.65,\ \lambda = 0.1397,\ \Gamma = 0.452 \qquad (\gamma = \tfrac{7}{5})$$

$\gamma \rightarrow 1$

$$A = K,\ \lambda = \Gamma^2$$

Both λ and Γ increase monotonically with K and coefficients B_4, B_5, B_6, and B_7 always remain small as compared to one. Since the frequency-dependent terms all include the factor k^2, they are all negligibly small for small values of k. It can be shown that for $k < 0.3$ and regardless of the values of K, x, and x_0, the frequency-dependent terms are almost always smaller than 1% of the corresponding frequency-independent terms, except for some extreme cases, where the latter terms themselves are very small.

In the double Newtonian limit, $\gamma \rightarrow 1$ and $K \rightarrow \infty$, we have

$$\Gamma = \lambda = 1,\ C_n = \tfrac{1}{2} \qquad (n = 0,1,2,3)$$

$$B_m = 0 \qquad (m = 0, 1, \ldots, 7)$$

However, the unsteady pressure is not necessarily zero, since for these conditions Eqs. (2) and (3) become indeterminate. Then, the value of $\bar{p}_{osc}$ has to be determined from a separate study, e.g., one that would involve an expansion in terms of the Newtonian parameters ϵ and N as shown in paragraph 2.6 of Ref. 1.

Slender Wedge—Large Amplitude ($\gamma = \frac{7}{5}$ and $\frac{5}{3}$)

For amplitude of oscillation which is of the order of wedge semiangle θ_w, the pitching motion may be expressed as

$$\theta = \alpha\theta_w \cos\omega\bar{t} \qquad (0 < \alpha < 1) \tag{6}$$

where θ is defined positive in the same direction as before and again the oscillation is about zero wedge incidence. For low reduced frequencies, the pressure on the upper surface of a slender wedge can then be obtained from a paper by Kuiken[3] as

$$\bar{p} = \bar{p}_m(1 + R\bar{A}\xi + S\bar{A}\chi) \tag{7}$$

where $\bar{p}_m$ is given in Eq. (1a), $\xi = \omega\bar{x}/U_\infty$ is the local reduced frequency parameter, $\chi = \omega l/U_\infty$ is the reduced frequency parameter based on the distance l of the axis of oscillation from the leading edge, R and S are functions of K and γ which were tabulated in Ref. 3, and

$$\bar{A} = \alpha \frac{[(K^2 - 1)/(2K)] \sin\omega\bar{t}}{\{[(K^2 - 1)/(2K)]^2[1 - \alpha \cos\omega\bar{t}]^2 + 1\}^{1/2}} \tag{8}$$

Using the procedure of curve-fitting, functions R and S of Ref. 3 can be expressed as analytical functions of the form

$$R \text{ (or } S) = (aK^4 + bK^2 + c)/(K^4 + dK^2 + e) \tag{9}$$

which agrees with Tables 1 and 2 of Ref. 3 within better than the last digit of the tabulated values if the constants in Table 1 are used.

As discussed in Ref. 3, Eq. (7) may be valid for general wedge motion if ($\alpha \cos\omega\bar{t}$) and ($\alpha \sin\omega\bar{t}$) in Eqs. (6) and (8) are replaced by functions $[1 - f(\omega\bar{t})]$ and $[\partial f(\omega\bar{t})/\partial(\omega\bar{t})]$, respectively, as long as $f(\omega\bar{t})$ is of the order of unity.

Slender Cone—Small Amplitude ($\gamma = \frac{7}{5}$)

Consider a slender cone, with the cone semiangle θ_c, which performs either a pitching oscillation

$$\theta = \theta_0 e^{i\omega\bar{t}} = \tau\alpha e^{ikt} \qquad (\theta \ll \tau)$$

about its vertex, or a plunging oscillation

$$h = h_0 e^{i\omega\bar{t}} = \tau\alpha e^{ikt} \qquad (h \ll \tau)$$

Expressing again the surface pressure on the oscillating cone as the sum $\bar{p} = \bar{p}_m + \bar{p}_{osc}$, we obtain, from the hypersonic small-disturbance analysis of Ref. 1 for the pitching cone,

$$\bar{p}_{osc} = \bar{p}_m\alpha e^{ikt} \cos\psi[(\tilde{P}_{1.0} + x^2\tilde{P}_{1.2} + x^4\tilde{P}_{1.4}) + ix(\hat{P}_{1.1} + x^2\hat{P}_{1.3} + x^4\hat{P}_{1.5})] \tag{10}$$

and for the plunging cone,

$$\bar{p}_{osc} = \bar{p}_m\alpha e^{ikt} \cos\psi[x(\tilde{P}_{1.1} + x^2\tilde{P}_{1.3} + x^4\tilde{P}_{1.5}) + i(\hat{P}_{1.0} + x^2\hat{P}_{1.2} + x^4\hat{P}_{1.4})] \tag{11}$$

where $\bar{p}_m = \bar{p}_\infty\gamma K^2 p_0$ is the steady mean pressure on the cone, p_0 is the steady dimensionless surface pressure, which is a known function of γ, K, and of the steady dimensionless surface density ρ_0, and ψ is the azimuthal coordinate, such that $\psi = 0$ at the top of the leeside of the cone when $\theta > 0$. Functions $\tilde{P}_{1.0}$, $\hat{P}_{1.1}$, . . . , etc. describe pressure perturbations at the surface and can be obtained, for a given combination of K, γ, and k, from the analysis of Ref. 1, which involves a numerical solution of a large number of simultaneous differential equations.

In each of the round brackets in Eqs. (10) and (11), the first term represents the perturbation of the lowest order in reduced frequency k, whereas the second and the third terms contain an additional factor k^2, as compared to the first term. Since in hypersonic stability analysis we always have $k \ll 1$, a very useful first approximation may be obtained by neglecting these higher-order terms. Rewriting Eqs. (10) and (11) and introducing the oscillatory variables θ and h and their time derivatives (indicated by dots), the pressure components at the surface of the oscillating cone are

$$\bar{p}_m = \bar{p}_\infty\gamma K^2 R_0$$

$$\bar{p}_{osc} = \bar{p}_\infty\gamma K^2 \cos\psi \left[R_1 \frac{\theta}{\theta_c} + x\frac{c\dot{\theta}}{U_\infty\theta_c} R_2\right] \text{(pitching)} \tag{12}$$

† Correcting a misprint, the left-hand side of that equation should read $\bar{p}_p/(\rho_2 a_2 U_2)$. In this connection, an error was discovered also in Eqs. (9) and (11) of Ref. 6, which should be, respectively,

$$\left(\frac{p_{inv}}{p_\infty}\right)_{U,L} = \frac{2\gamma\bar{K}^2 - \gamma + 1}{\gamma + 1} \mp \frac{2\gamma\bar{K}^3}{\bar{K}^2 + 1} M_\infty\theta_{uns}$$

$$F_0 = 4\bar{K}^3/(\bar{K}^2 + 1)$$

In a remark after Eq. (13) in Ref. 6, this makes function F_0 identical to the previously used function $F(M_\infty\theta_w)$ in both the lower limit (weak shock) and the upper limit (strong shock). Although the difference between the correct value of F_0 and that initially given in most cases is small, Fig. 2 in Ref. 6 should no longer be used.

or

$$\bar{p}_{osc} = \bar{p}_\infty \gamma K^2 \cos\psi \left[x \frac{c^2\ddot{h}}{U_\infty{}^2\theta_c} R_3 + \frac{c\dot{h}}{U_\infty\theta_c} R_4 \right] \text{(plunging)} \tag{13}$$

where $R_0 = p_0$, and, letting $P \equiv p_0\theta_c/\tau$,

$$R_1 = P\tilde{P}_{1.0},\ R_2 = \hat{P}P_{1.1}/k$$
$$R_3 = -P\tilde{P}_{1.1}/k^2,\ R_4 = \hat{P}P_{1.0}/k = R_1 \tag{14}$$

Since the analysis of Ref. 1 is based on the hypersonic small-disturbance theory, all the pressure ratios are functions of only γ, K, and of the oscillatory variables normalized by cone semiangle, as they should.

Numerical values of ρ_0 and of the 12 pressure perturbation functions appearing in Eqs. (10) and (11) have been calculated by McIntosh for $\gamma = 1.4$, $k = 0.5$ and 1.0 and for 12 values of the hypersonic similarity parameter K ranging from 1.04 to 100. Curve-fitting these numerical results, each of the functions R_0, R_1, R_2, and R_3 may be represented as a polynomial in K^{-2}, namely,

$$R_m = -\sum_{n=0}^{6} a_{mn}K^{-2n} \qquad (m = 0,1,2,3) \tag{15}$$

where

$$a_{00} = -0.8747,\ a_{01} = -0.0909,\ a_{02} = 0.1049$$
$$a_{03} = -0.5608,\ a_{04} = 1.6659,\ a_{05} = -2.1070$$
$$a_{06} = 1.0420,\ a_{10} = 1.7060,\ a_{11} = -1.8456$$
$$a_{12} = 0.4287,\ a_{13} = 0.4196,\ a_{14} = -1.8153$$
$$a_{15} = 2.3851,\ a_{16} = -1.1774,\ a_{20} = 2.3316$$
$$a_{21} = -3.8403,\ a_{22} = 3.7825,\ a_{23} = -2.7061$$
$$a_{24} = -0.2412,\ a_{25} = 2.3776,\ a_{26} = -1.5319$$
$$a_{30} = 0.3128,\ a_{31} = -0.9976,\ a_{32} = 1.6810$$
$$a_{33} = -1.5845,\ a_{34} = 0.8356,\ a_{35} = -0.0520$$
$$a_{36} = -0.1596$$

By comparing R_m obtained from Eq. (15) with the corresponding results of the numerical analysis, the maximum relative error introduced by the curve-fitting in the range of $1.04 < K < 100$ may be calculated. By comparing the maximum values of the second terms in parentheses in Eqs. (10) and (11) to the corresponding first terms and neglecting the remaining terms altogether, the maximum relative (percentage) errors due to omission of higher terms may be determined and are given in Table 2. The largest errors due to curve-fitting occur for low values of K, where functions R_m undergo their largest variations with K. For $K > 1.3$ the relative errors decrease with increasing K.

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